Roll. No. | | | | | | | | | | |

# ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

## B.E (Full Time) END - SEMESTER EXAMINATIONS, MAY/JUNE - 2024

## Computer Science and Engineering - IV Semester

## MA6201 - Linear Algebra (Regulations 2018 - RUSA)

Time: 3 Hours | Maximum: 100 marks

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| CO1 | Perform linear transformations and write down the matrix representing a linear transformation. |
| CO2 | Find the Gram-Schmidt orthogonalization of a matrix. |
| CO3 | Determine the rank, determinant, eigenvalues and eigenvectors, diagonalization, and different factorizations of a matrix. |
| CO4 | Solve a linear system of equations using direct and iterative methods. |
| CO5 | Solve Eigen value problems. |
| CO6 | Formulate linear equations for real life problems and solve them. |

**BL - Bloom's Taxonomy Levels:** L1 - Remembering; L2 - Understanding; L3 - Applying; L4 - Analysing; L5 - Evaluating; L6 - Creating.

Answer ALL questions.

**PART - A (10 × 2 = 20 Marks)**

| Q.No | Questions | Marks | CO | BL |
|---|---|---|---|---|
| 1. | Is the set $\{f(x) \in P_3(\mathbb{R}) : f(1) = 1\}$ a subspace of $P_3(\mathbb{R})$? Justify the answer. | 2 | CO1 | L2 |
| 2. | State true or false with explanation. If $V$ is a finite dimensional vector space over a field $F$, then all the bases of $V$ have the same number of elements. | 2 | CO1 | L1 |
| 3. | Let $\beta = \{1, x+4, 8x^2+6, 9x^3\}$ be an ordered basis for $P_3(\mathbb{R})$. If the coordinate vector of $f(x)$ relative to the basis $\beta$ is $[f]_\beta = [4, 6, -9, 2]^T$, then find $f(x)$. | 2 | CO1 | L4 |
| 4. | Explain whether the matrix $\begin{pmatrix} -4 & 0 & 0 \\ 2 & 5 & 0 \\ -1 & 1 & 3 \end{pmatrix}$ is diagonalizable? | 2 | CO3 | L3 |
| 5. | Consider the vector space $\mathbb{R}^2$ and let $x, y \in \mathbb{R}^2$. Determine whether $\mathbb{R}^2$ with the function $< x, y >= x_1y_2 + x_2y_1$ is an inner product space. | 2 | CO2 | L1 |
| 6. | Prove that $x$ and $y$ are orthogonal if and only if $\|x+y\|^2 = \|x\|^2 + \|y\|^2$. | 2 | CO2 | L1 |
| 7. | Write any two differences between the direct methods and iterative methods for solving the system of linear equations. | 2 | CO4 | L2 |
| 8. | Write the formula of Gauss-Seidal method for the system of linear equations with three unknowns. | 2 | CO4 | L2 |

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| 9. | Find the dominant eigenvalue and the corresponding eigenvector of the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ by using power method. Take the initial approximation as $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. | 2 | CO5 | L2 |
| 10. | Find the singular values of the matrix $\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. | 2 | CO6 | L1 |



Answer ANY EIGHT questions

**PART - B (8 × 8 = 64 Marks)**

| Q.No | Questions | Marks | CO | BL |
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| 11. | Check whether the set $S = \{x^3, 2x^2, x, 0\}$ is linearly independent or not in $P_3(\mathbb{R})$? Does $S$ generate $P_3(\mathbb{R})$? Explain. | 8 | CO1 | L3 |
| 12. | Let $V$ be a vector space and $u_1, u_2, \ldots, u_n$ be distinct vectors in $V$. Then prove that $\beta = \{u_1, u_2, \ldots, u_n\}$ is a basis for $V$ if and only if each $v \in V$ can be uniquely expressed as a linear combination of vectors of $\beta$. | 8 | CO1 | L4 |
| 13. | The matrix of the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ relative to the ordered basis $\{(0,1,1)(1,0,1)(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0)(0,1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 1 \end{pmatrix}$. Find the linear transformation and the matrix of $T$ relative to the ordered basis $\{(1,1,0)\}(1,0,1)(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,1)(0,1)\}$ of $\mathbb{R}^2$. | 8 | CO1 | L4 |
| 14. | Let $T$ be the linear operator on $P_2(\mathbb{R})$ defined by $$T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2.$$ Test $T$ for diagonalizability. | 8 | CO3 | L3 |
| 15. | Determine the eigenspace corresponding to each eigenvalues of the matrix $\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$. Also, find its dimensions. | 8 | CO3 | L5 |
| 16. | Prove that the set $\{\frac{1}{\sqrt{5}}(1,2), \frac{1}{\sqrt{5}}(2,-1)\}$ is an orthonormal basis for $\mathbb{R}^2$ with respect to the standard inner product. | 8 | CO2 | L3 |
| 17. | Apply Gram-Schmidt orthogonalization process to obtain an orthonormal basis for the subspace of $\mathbb{R}^3$ geberate by $\{(1,0,1),(1,0,-1),(0,3,4)\}$ with respect to the standard inner product. | 8 | CO2 | L5 |
| 18. | Apply Gauss Jordan method to solve the system of linear equations $x - y + z = 1$, $-3x + 2y - 3z = -6$, $2x - 5y + 4z = 5$. | 8 | CO4 | L4 |

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| 19. | Solve the system of equations $x + y - z = 4$, $x - 2y + 3z = -6$, $2x + 3y + z = 7$ by LU decomposition method. | 8 | CO4 | L4 |
| 20. | Use SOR method to solve the system of equations $2x - 8y + z = -5$, $3x + y - z = 3$, $x - 2y + 9z = 8$. | 8 | CO4 | L4 |
| 21. | Find all the eigen values and eigenvectors of the matrix $\begin{pmatrix} 2 & 3 & \frac{1}{\sqrt{2}} \\ 3 & 2 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 5 \end{pmatrix}$ using Jacobi rotation method. | 8 | CO5 | L5 |
| 22. | Find the QR decomposition of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$. | 8 | CO6 | L5 |

## PART - C (2 × 8 = 16 Marks)

| Q.No | Questions | Marks | CO | BL |
|---|---|---|---|---|
| 23. | Let Define the linear transformation $T : P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ by $T(f(x)) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}$. Verify the dimension theorem. | 8 | CO3 | L3 |
| 24. | Find the singular value decomposition of the matrix $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$. | 8 | CO6 | L5 |